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Tail Risk Hedging: Contrasting Put and Trend Strategies

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Abstract

We summarize key research findings on risk-mitigating strategies and offer an overview of the strengths and weaknesses of regular index put buying ("Put") and multi-asset trend following ("Trend") as tail hedges. The two biggest questions we address are: (1) What is the long-term average return or cost, and (2) How reliable and efficient is the hedge in equity market tail events? We present *empirical* answers and discuss the *economic rationale* for each question. The common view that Put costs more but is a more effective tail hedge contains a kernel of truth but does not capture the full story. We will give a more nuanced picture, including practicality for investors, but in the end show the cost advantage favors Trend over Put.

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Introduction

The sharp market fall and speedy recovery during the eventful first half of 2020 has kept tail risk hedging topical: Investors have both fresh memories of a painful loss and renewed fears of a repeat. In this paper we summarize key research findings on risk-mitigating strategies[1] and try to offer a balanced overview of the strengths and weaknesses of direct and indirect tail hedging strategies.

For brevity, we represent direct tail hedges with long out-of-the-money (OTM) index put strategies **("Put")**, and indirect tail hedges with multi-asset-class trend-following strategies **("Trend")**.[2] This article is broken up into two major sections to address: (1) What is the long-term average return or cost of these strategy types; and (2) How reliable and efficient are they as hedges in equity market tail events? Within each section, we A) present *empirical* answers and, B) discuss the *economic rationale*. A third section addresses real-world investor behavior.

The common view that Put costs more but is a more effective tail hedge contains a kernel of truth but does not capture the full story. We will give a more nuanced picture, including practicality for investors. In the end, we find a generally stronger case for Trend than for Put, except for investors who focus primarily on fast market drawdowns.

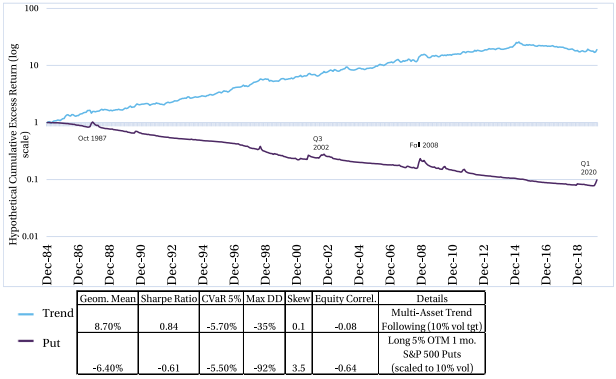
What Is the Long-term Average Return or Cost of Put and Trend Strategies?

Empirical Evidence

For the empirical answer, Figure 1 contrasts the persistently negative performance of Put (here, a strategy of buying a 5% OTM one-month index put every mid-month and rolling into a new put at expiry)[3] with the overall positive return of multi-asset Trend[4] over 35 years. Many investors fear sharp market declines, so it is not surprising that option-based protection against such events has very high cost. It is more surprising that Trend has been able to combine positive long-run returns (even if muted in the 2010s) with strong performance in most market tail events. Put did make timely gains in sharp bear markets but spent those gains soon after by buying more expensive puts. The two series are scaled to have roughly comparable standalone risk (as measured by volatility or, more importantly, conditional value at risk), but the different signs of average returns are independent of scaling.

Other studies have discussed the strong Trend

Figure 1: Contrasting Long-Term Performance of Put and Trend Strategies, January 1985 – March 2020



	Geom. Mean	Sharpe Ratio	CVaR 5%	Max DD	Skew	Equity Correl.	Details
Trend	8.70%	0.84	-5.70%	-35%	0.1	-0.08	Multi-Asset Trend Following (10% vol tgt)
Put	-6.40%	-0.61	-5.50%	-92%	3.5	-0.64	Long 5% OTM 1 mo. S&P 500 Puts (scaled to 10% vol)

Notes: The Hypothetical Put strategy is a backtest which involves buying a 5% out-of-the-money one-month put on the S&P 500 index (pre-1996 on the S&P 100) at mid-month and rebalancing into a new put at expiry. Put returns are expressed as a percentage of the underlying index NAV, gross of trading costs and fees. For comparability, the series is scaled to 10% volatility based on the 6% volatility of the unlevered return over the full sample, implying a leverage of 1.67. The hypothetical Trend return is a backtest, gross of fees, net of estimated transaction costs. The strategy applies trend following at one-, three- and twelve-month windows in four asset classes and targets overall portfolio volatility of 10%. Both Put and Trend returns are in excess of cash (US three-month LIBOR) or using self-financed futures/forwards. Appendix describes the hypothetical Trend Following strategy.

Source: AQR, Bloomberg, Commodity Systems Inc., and Option Metrics.

performance over the past century and its weaker performance over the last decade in many articles and the persistent long-term cost of Put in others.[5] Since the latter result may be more controversial, we expand on it.

In particular, Figure 1 portrays clear evidence of historical negative returns of Put but the interesting question is whether we would expect history to repeat itself and long-put strategies to continue to lose money in the future. The answer depends on the reason for past persistent losses. As we will discuss in the theory section, option prices may embed various risk premia, partly related to puts' insurance characteristics. (Such risk premia are often inferred from the empirical fact that option prices imply volatilities and negative skewness that tend to systematically exceed subsequent realizations and thus likely market expectations. This pattern is the proximate cause for negative put returns.)

Option prices reflect risk-neutral expectations, which in turn reflect some blend of the market's real-world expectations and required risk premia. The mere fact that all options do not have the same "Black-Scholes implied volatilities" across strikes reveals that markets do not discount normally distributed future returns. Index option pricing has always had a smile pattern, higher implied volatilities for OTM options discounting fat tails. After the 1987 Crash, the smile became an asymmetric skew or smirk (deep-OTM puts having highest implied volatilities). Option markets "know" that actual stock returns are fat-tailed; for option *strategies* the key question is whether this is fully priced (or more) in option prices.[6]

An alternative view states that option prices reflect fair market expectations of very rare but impactful events (which may not materialize in a given sample), suggesting that the negative returns we document are specific to the sample and options that we study. We next evaluate this critique in more detail and conclude that given the length and nature of the sample period, as well as other data in our disposal, the result of negative returns to long-put strategies is likely a robust one.

Three important follow-up questions are: (i) Do we have enough data?; (ii) What about robustness to other Put strategies, such as deeper OTM puts?; and (iii) Can active tail hedge managers do better?

i. **Do we have enough data?** A study of rare events requires very long histories, so one can debate whether 35 years is enough. It would be nice to have even more data, like the century or more we have on Trend, but index option markets developed only in the 1980s. Two potential ways to address if our sample should be representative of go-forward expectations are to ask if the sample period was exceptionally adverse to Put (by being too benign for markets) and if "out-of-sample" evidence from other markets is consistent with what we document for the S&P 500 index options.

   - Over the 35 years or so where we have good index option data, OTM put prices were high enough to give negative returns each decade—despite big market events like 1987 (the biggest daily crash in history), 1998 (Russia/LTCM crisis), 2001 (9/11), 2008 (Lehman), 2020 (Covid), as well as a recession roughly every decade and two bear markets where the market lost roughly half of its value. This was *not* an uneventful sample period.
   - There is evidence of long-option strategies underperforming in other countries and asset classes.[7]

ii. **What about robustness to other put strategies?** The broad pattern in Figure 1 is robust to every specification of passive put buying that we tested. In particular, we see very similar patterns regardless of our choice of maturity or moneyness. Pre-1996 data is scarce on deeper or longer-dated OTM

options, but we present results for a range of strategies from 1996. For example, in contrast to our baseline specification (buying a 5% OTM one-month put, and re-initiating a new 5% OTM one-month put after expiration), a simple 20% OTM one-year put-buying strategy involves protecting wealth at 20% below the prevailing market level each June or December. In addition, every six months we roll the then six-month put into a new one-year put in order to maintain exposure to longer-dated puts.

Figure 2 shows that all the series studied share the same pattern of persistent losses, interspersed by temporary spikes. The other series lose less over time than the baseline Put strategy, but mainly because they are less risky (whether measured by volatility, 1% or 5% VaR, 1% or 5% cVaR, equity beta or volatility exposure).[8] Tail hedgers could apply higher leverage on less risky strategies, thus possibly resulting in comparable cumulative losses.

iii. Can active tail hedge managers do better?

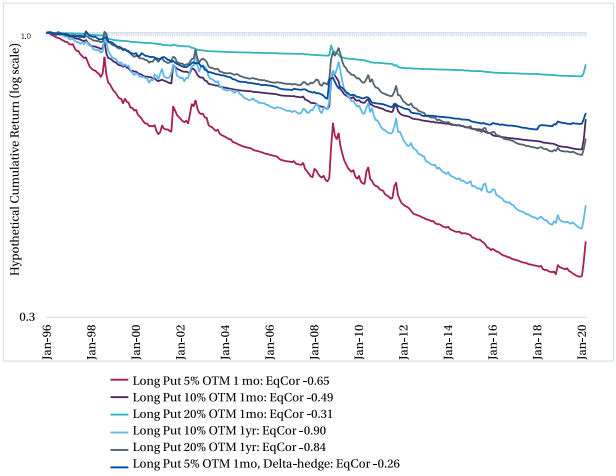
It is certainly possible. But this should not be taken for granted and might come at the expense of the tail hedging ability. It is hard to identify successful active managers in advance, and it is correspondingly hard to distinguish the role of luck versus skill among the ex-post winners—and even harder for strategies focused on rare events.

This is why the Put series shown here is a useful benchmark, even if its construction is straightforward or offers too unattractive reward-for-risk for investors to consider it. It is close to contractually specifying the protection floor and its performance/cost can be tracked over multiple decades.

Turning to live performance, the CBOE Eurekahedge Tail Risk Index, a peer index of tail risk managers, has the longest history. Since its inception in 2008 it has earned a return between those of the Trend and Put series (around −2% per annum, but −8% per annum during the bullish 2010s). Since the index was created in 2015, survivorship bias may have boosted returns between 2008 and 2014. Some managers have outperformed this index, including some focusing on options and some using indirect hedges, yet the 2010s was a challenging period for most of them.

One possible way to outperform Put is through tactical timing. However, predicting crashes may be even more difficult than market timing. For example, Israelov and Nielsen (2015) and Israelov and Tummala (2018) show using option market data that neither calm markets nor rising volatility have

Figure 2: Cumulative Returns of Six OTM Put Strategies, February 1996 – March 2020



Notes: Unlevered option returns are expressed as percentage of underlying index NAV, gross of trading costs and fees, excluding cash. (That is, these are "constant notional" put returns, using as the denominator the S&P 500 index value when the put was first traded.) One-month puts are rolled every month, one-year puts every June and December. The delta-hedged puts are hedged using the Black-Scholes options pricing model and their implied volatilities. EqCor is the correlation with S&P 500 returns over the full sample period Feb 1996 to March 2020.

Source: AQR, Bloomberg and Option Metrics.

been empirically helpful timing signals for turning on the tail hedge. Most proponents thus recommend "always-on" tail hedging and debate how it is best done. Such alpha-seeking tail hedge selection is manager-specific and often discretionary, but Bhansali (2014) lists several techniques which may help achieve a lower long-run cost and/or better tail performance than a benchmark of static put rolling.[9] Using a constant tail hedge budget may also reduce long-run cost (due to an implicit value tilt), while varying the degree of protection. Similarly a strategy that partially or fully finances the cost of puts, such as a collar, may have less of a long-term performance headwind (but at the expense of expected hedging efficacy).

Economic Rationale

What about theory? This is especially important since we have only a few decades of index option data. For Put or any strategies that try to hedge large equity market losses, the very risk many investors most dislike, it's rational for the risk premium to be negative. At the heart of virtually all asset pricing models is the idea that investors require and earn positive long-term rewards for investments that deliver bad returns in bad times (intuitively, recessions and bear markets). Conversely, investors should accept low or even negative long-term returns for safe-haven assets and for strategies that provide good performance in bad times—just as insurance buyers are willing to pay an extra premium for avoiding the worst outcomes. This is an intuitive concept: long OTM puts are expensive because they provide a useful insurance service for typical portfolios.

The simplest theories refer to the negative equity market beta of long puts warranting a negative premium. Other theories add a negative premium for their long volatility exposure (the volatility risk premium), or a skewness or a jump/gap risk premium, due to investor beliefs or preferences.[10] Some even point to "crash-o-phobia" to explain the particular richness of deep OTM puts since 1987.[11] On the other side, there is the argument that carry-seeking preferences can make puts cheaper.

In contrast to Put, Trend is primarily a return-seeking strategy with tail hedging benefits being a useful by-product. Selling "risk-on" positions after they have suffered can be profitable if such market moves are persistent (as they have been historically, on average). The behavioral underpinnings of trend following—investors underreacting to public news and yet overreacting to (i.e., extrapolating) recent price changes—suggest that market moves indeed have some tendency to be gradual and protracted. This observation also identifies sudden directional turns as a key vulnerability for Trend and a relative edge for Put. The vulnerability of Trend to sharp, discontinuous market turns echoes the criticism toward portfolio insurance strategies after the 1987 Crash. It is no coincidence that many dynamic loss or risk control strategies—stop-loss rules, drawdown control, portfolio insurance, volatility targeting, value-at-risk management—share with Trend the pattern of selling risky assets after market weakness or after rising risk. Thus, they inherit some trend-following features, as documented going back to Fung and Hsieh (2001). Similarly, Hamill, Rattray, and Van Hemert (2016) show the payoff of a trend-following strategy resembles that of a long straddle (roughly, a mild U-shape) but will miss gapping market moves. Finally, it is fair to ask if Trend's empirical blend of positive long-run returns and tail hedging ability is "too good to be true," and suspect that Trend's future performance cannot be as compelling on both fronts.

How Reliable And Efficient Is the Hedge Provided by Put and Trend in Equity Market Tail Events?

Empirical Evidence

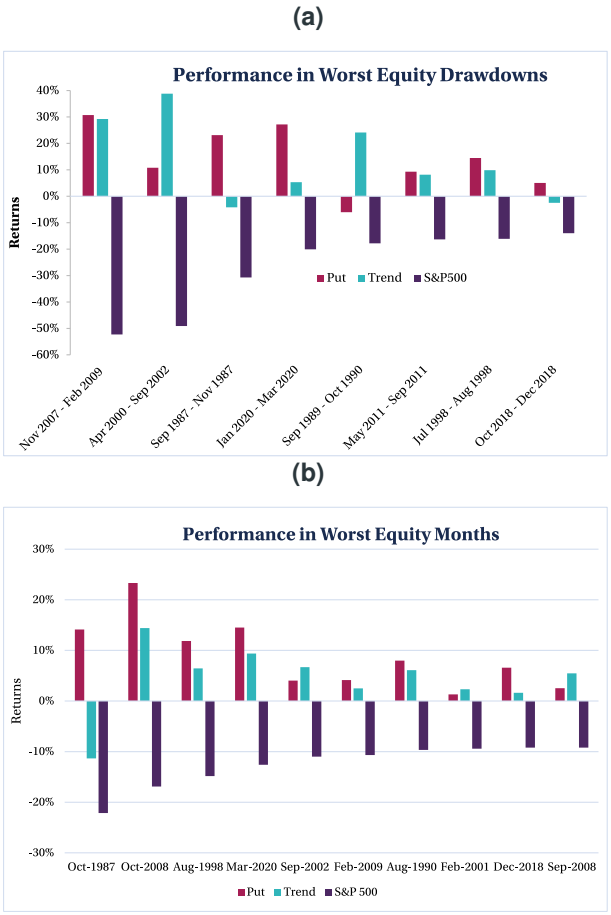
The first section showed that Put was a costly strategy over the long term—yet it may be worth it if it provides particularly good tail hedging benefits. Figure 3 assesses the tail performance of both Put and Trend, contrasting slow and fast tail events. Figure 3a examines the worst peak-to-trough drawdowns of the S&P 500 index in recent decades (which range between two and thirty months in length), while Figure 3b focuses on its worst single months.

The main message from Figure 3 is that both Put and Trend performed well in most tail events, whether fast or slow. They have done better than many other tail hedge candidates, such as Treasuries or gold.[12] Studying "hit rates" (frequency of positive returns), Put was profitable in seven of eight slow events and all ten fast events, while Trend was profitable in six of eight slow events and nine of ten fast events. The average returns were similar for both in slow events but higher for Put in fast events.

If we study even longer multi-year horizons, Put's cost drag dominates and hurts its performance. McQuinn, Thapar, and Villalon (2021) focuses on fast versus slow protection, where both logically and empirically Put has a relative edge in fast market drawdowns, while Trend and other strategies with positive expected returns have an edge in slower ones. The report argues that hedging against slow multi-year drawdowns is more important than hedging fast drawdowns, at least for those who claim to be long-horizon investors.

The most surprising result in Figure 3 is that Trend was up in nine out of the ten worst months (all but the perhaps most famous counterexample of

Figure 3: Hypothetical Tail Event Returns of Put and Trend During (a) Slow and (b) Fast Equity Drawdowns



Notes: The hypothetical Put strategy is a backtest which involves buying a 5% out-of-the-money one-month put on the S&P 500 index (pre-1996 on the S&P 100) at mid-month and rebalancing into a new put at expiry. Put returns are in excess of US three-month LIBOR and are expressed as a percentage of the underlying index NAV, gross of trading costs and fees. For comparability, the series is scaled to 10% volatility based on the 6% volatility of the unlevered return over the full sample, implying a leverage of 1.67. The hypothetical Trend return is a backtest, gross of fees, net of estimated transaction costs, and constructed to be dollar neutral. The strategy applies trend following at one-, three- and twelve-month windows in four asset classes and targets overall portfolio volatility of 10%.

Source: AQR, Bloomberg, Commodity Systems Inc. and Option Metrics.

October 1987). This result may be partly chance, or it might suggest that many worst-months don't come out of the blue but rather follow earlier trouble (allowing Trend to position itself "risk-off" in time). This is just what we find.

- Since this empirical result has not been spelled out in earlier literature, we list here the ten worst months for the S&P 500 and note which month of its broader market drawdown it represents (for example, Oct-1987 was the second month in a three-month drawdown). Oct-1987: 2/3, Oct-2008: 12/16, Aug-1998: 2/2, Mar-2020: 3/3, Sep-2002: 30/30, Feb-2009: 16/16, Aug-1990: 12/14, Feb-2001: 11/30, Dec-2018: 3/3, Sep-2008: 11/16.

- Interestingly, none of the worst ten months was the first one within its broader drawdown episode, whereas half of them were the last month within the episode. The latter result likely reflects the "Fed put" (sharp market falls can trigger supportive central bank action and thus stop the market from falling further).

- It is fair to ask if Trend's success in tail events may reflect overfitting to historical episodes. Trend is, after all, a backtest. To address this concern, we studied both a simpler strategy than Trend (only following twelve-month trends, thus hardly fitting to crash experiences) and live peer indices (the BarclayHedge CTA index since 1980s and the SG Trend index since 2000; the latter contains purer trend-followers, while the former reflects CTAs' tendency to include carry and other strategies). In both cases, we found almost as good empirical tail performance as for Trend. These results are available upon request.

- There is of course no guarantee that the next market drawdown won't be the rare sudden shift from a "risk-on" to "risk-off" environment. Although the behavioral underpinnings of trend-following suggest gradual evolution is more likely, we should not rule out sudden exogenous shocks.

Economic Rationale

We can also ask logically about the *reliability* of each strategy (how often should it earn positive returns, and if a given wealth floor is specified, how reliably is it protected?) as well as about its *efficiency* (the convexity of payoffs in tail events, the scalability of protection for large institutions).

i. **Reliability:** Since Put is virtually designed to deliver tail protection, while Trend's tail hedging benefits are less direct, it is fair to expect better reliability from Put. Moreover, the 1987 experience taught market participants that in gapping market falls, option-based protection strategies are more reliable than dynamic strategies, such as portfolio insurance, which depend on the ability to trade continuously.

   In practice though, Put has sometimes disappointed these high expectations, while Trend has surprised on the upside. Why might this be?

   Even for fast market crashes, Israelov (2017) shows that Puts offer somewhat patchy protection unless the actual market decline aligns fortuitously well with the maturity and strike price of the put. In practice, option-based tail hedgers often combine multiple option maturities and strikes to reduce such path-dependence. Trade-offs between reliability and both cost saving and convexity enhancement will sometimes lead to compromises in protection. Trend is vulnerable to sharp market turns, but as noted, the worst falls have usually occurred later in a bear market, allowing trend followers often to benefit even from gapping moves.

   Slower market declines, such as the 2000–02 Tech Bust or the Nikkei 225 decline since the early 1990s, are even more problematic

for Put strategies. It is possible that the put strikes are never or rarely reached in a gradual bear market where the cumulative fall nonetheless reaches 50%. The fact that put prices tend to rise amid such environments compounds the problem. Trend strategies fit better with such markets. We simply do not know whether the next drawdown will be of the gradual variety (say, a slow decay caused by some combination of high market valuations, reduced central bank support, and shifts towards deglobalization, anti-market sentiment, buffers-instead-of-efficiency, and ageing populations), or a fast one (e.g., driven by an exogenous shock like 9/11 or Covid-19).

Further, one's actual portfolio may not match the available hedging assets. For example, if the S&P 500 index falls much less than one's actual equity portfolio (say, a U.S. small-value portfolio or a non-U.S. equity portfolio), the S&P puts will provide only partial protection. That said, Trend and other indirect hedge strategies tend to have even more of such basis risk than Put due to their multi-asset nature.

Then there is the question of whether the tail insurance provider will be around after a crash event. Counterparty risk is an important consideration for financial catastrophe insurance. Tail insurance providers that only use long-option strategies may claim to be safer due to options' limited downside, but they too need to be able to collect their gains from their counterparties after a crash to pay their clients. Separately, there is the question whether *the tail insurance buyer* will still be around (paying those costly tail insurance fees) when the next crash event materializes; we will return below to the real-world danger of investor impatience.

ii. **Efficiency:** Convexity refers to the asymmetry or nonlinearity whereby a small position in a tail hedge can "move the needle" and provide gains that offset a meaningful part of losses caused by the equity market fall. Buying or selling the underlying asset gives linear exposures. ATM options and most indirect hedges give some convexity. Only deep OTM puts (or spread positions) can give extreme convexity, such as 5–10× payoffs on a small annual tail hedge allocation. The flipside is that most deep OTM puts will expire worthless.

How valuable is such convexity to investors? It depends on their risk preferences. Standard utility functions (i.e., without a wealth floor) often imply preferences for positive skewness, which should translate to a negative risk premium for strategies (like Put) that have this characteristic. Moreover, for investors with highly asymmetric risk preferences, such as no tolerance for calendar-year losses below 20%, this preference may be stronger still.[13] If the market has many investors with such preferences, deep-OTM puts (with valuable convexity properties) should be priced to have a costly insurance premium.

Extreme convexity is a key advantage of option-based tail hedges. Both Put and Trend strategies can be designed to be more convex. There are inevitable trade-offs between cost, reliability and convexity (e.g., a deeper OTM Put gives a lower protection floor which pays off more rarely, but it requires a smaller outlay and offers more convexity). In practice, even the levered OTM puts we study above did not exceed 50% monthly return in the worst equity months (nor did the Eurekahedge Tail Risk Index), but some managers have achieved this. Thus, manager-specific alpha is required for greater returns during these periods.[14]

Scalability is another aspect of efficiency. While the S&P 500 index option market may be the most liquid option market, it does not have the depth of, say, the index futures market. Trading costs can be high, especially

for the OTM options as a percentage of the outlay. Market participants suggest that while option markets offer capacity to insure medium-sized institutional portfolios, transaction costs can be considerable for investors with very large portfolios.

How Impatience Can Make the Investor Experience Even Worse

Looking at the negative standalone returns, long Put strategies are clearly unattractive. Yet the actual investor experience may be even worse because the episodic nature of "jackpots" makes it common for investors to chase these strategies after one jackpot and give up before the next one materializes.

Tail hedging strategies should ideally not be viewed standalone but in conjunction with the portfolio they are supposed to hedge (or insure, or protect against the worst tail outcomes). But most real-world investors, even those who believe in portfolio perspective and patience, (i) cannot fully resist line-item thinking, (ii) mainly judge performance *after they invested* into a fund (at most giving partial credit for earlier wins such as 2008, even if they are part of a public audited track record), and (iii) will find it difficult to stick with a strategy which underperforms more than five years.[15]

In reality, the standalone performance of option-based tail hedges may have involved a decade or more (2009–19) of not just underperforming, but of spending most of the capital allocated to them. Whether it is fair or not, the high bleeding costs of the Put strategy make it less likely that investors will even achieve the long-run returns or tail rewards depicted above; capitulating before the protection event occurs is an all-too-plausible outcome. This return-and-patience advantage is another issue that favors Trend over Put and more generally indirect tail hedges over direct (option-based) tail hedges.

We should qualify the return advantage of Trend over Put. First, Trend also had a disappointing decade in the 2010s, barely earning positive returns. More importantly, active option-based tail hedgers may seek to reduce the cost of hedging through timing or selection of tail hedges (though here too, such "alpha" may compromise protection characteristics). Really countering investors' impatience may require their taking an integrated view: With a credible tail hedge in place, investors can increase their equity allocation (either strategically—say, from 60/40 to 70/30—or opportunistically, using monetized tail hedge gains after a market fall). The hope is that investors then judge the tail hedge performance together with this higher equity allocation and not standalone. Bhansali (2014) calls this offensive risk management. Overall, tail hedge managers may thus reduce the actual and perceived cost drag of option-based tail hedging if they can convince investors of their alpha-generating abilities or of taking an integrated view.

Conclusions

We now weigh the variety of pros and cons discussed above. Unlike Trend, long Put strategies have had persistently negative returns despite, or perhaps because of, their gains during crashes (i.e., the cost for a valuable insurance service). This jibes with the balance of economic theory which would suggest a negative risk premium for insurance-like strategies. Yet, the documented return drag may be mitigated if the tail hedge allows an investor to take more equity risk and earn a premium for it, or if active tail hedge managers can offer "alpha" over Put.

Both Put and Trend strategies have good hit rates in most equity market drawdowns, with important differences. Put strategies have offered more reliable tail insurance than Trend in fast market drawdowns, especially in gapping markets like October 1987. The convexity of deep-OTM put strategies also makes it more plausible to fully

offset a large market drop; indirect tail hedges rarely offer as extreme payoffs.

That said, Trend does tend to make money in the largest equity market monthly drops, as they typically do not occur out of the blue. Moreover, Trend is better suited to slower, protracted bear markets. In these scenarios, Put is hampered by more sensitivity to the exact path of negative returns (e.g., a slow drawdown in which puts continuously expire out-of-the-money) and the general drag associated with paying the volatility risk premium being more relevant over longer horizons.

In our opinion, the long-term cost argument tips the scales in favor of Trend, unless investors are explicitly focused on hedging fast market drawdowns. This view is reinforced by the inevitable investor impatience during the dry spells when tail insurance costs are paid year after year before the tail event materializes. A good strategy is one that an investor can stick with; Put-based tail hedging on its own too often fails this test. Trend strategies do not offer as direct or explicit tail protection, but they have a strong empirical record, and investors likely have a better chance of sticking with Trend.

Tail risk hedging does not have to be an either/or decision: Even investors with strong preferences for Put-like payoffs might consider blending in some Trend-like strategies to facilitate a strategic allocation to portfolio risk mitigation, and to protect against more than just fast market falls.

Notes

1. See for example Ilmanen (2012, 2013), AQR Portfolio Solutions Group (2015, 2018), Asvanunt, Nielsen, and Villalon (2015), Hurst, Ooi, and Pedersen (2017), Israelov (2017), Israelov, Nielsen, and Villalon (2017), Nielsen, Thapar, and Villalon (2019), and McQuinn, Thapar, and Villalon (2021).

2. There are other ways to implement direct and indirect tail hedges (for more, see the papers referenced throughout), and there are many variants among Put and Trend strategies (for example, for Trend, see Hamill, Rattray, and Van Hemert (2016)), but we can address many important issues through this fundamental dichotomy.

3. Option-based tail hedging strategies sometimes use deeper OTM puts, say, 15–25% OTM puts (that is, protecting wealth at 15–25% below the current level). Since we want to include the 1987 Crash in our history, and option data before 1996 is limited, we first show evidence using the 5% OTM put, and later discuss other variants. Moreover, we only have access to S&P 100 index option data before 1996, but our evidence concurs with the Chernov, Broadie, and Johannes (2009) finding of broadly similar returns for 6% OTM puts in Oct 1987 and in Sep 2001 using the Berkeley database on S&P 500 index options.

4. "Trend" applies trend following not only on the S&P 500 or only on the equity asset class, but on dozens of assets in multiple asset classes: equity index futures, government bond futures, currencies and commodity futures (averaging one-, three- and twelve-month trends, and volatility-weighting between the constituent assets; see Hurst, Ooi, and Pedersen (2017) for details). Such breadth improves the Trend strategy's Sharpe ratio and, perhaps surprisingly, does not appear to hurt its equity market tail hedging ability (while improving its ability to hedge against other tail events such as rising bond yields or inflation rates). Historically, Trend benefited from risk-off positions in all or most asset classes during protracted equity bear markets as it involved shorting equities, buying duration, favoring anti-carry currencies, and buying gold against more cyclical commodities. In some faster bear markets, such as the first quarter of 2020, Trend actually lost money in equities, but gains

in other asset classes resulted in an overall positive return.

5. See, for example, Hurst, Ooi, and Pedersen (2017) and Babu et al. (2019) on Trend and Asvanunt, Nielsen, and Villalon (2015), and Israelov, Nielsen, and Villalon (2017) on Put.

6. Disentangling the expectations and risk premia components is hard. Could the tendency for implied volatilities to exceed realized ones reflect biased expectations instead of a volatility risk premium? How much does the asymmetry in implied volatilities reflect asymmetric return expectations (higher volatility in down-moves as markets tend to melt down, not up) versus asymmetric risk premia (such as skewness preference)? Any answers will be model-specific. Yet we can answer empirically whether the discounted view in option prices was excessive over a given sample period simply by studying realized option returns. This model-free approach allows us to bypass the twin modeling debates on what distributional assumptions are discounted in option prices and what the real-world dynamics are; we simply observe the "net" effect in realized option returns.

7. See Rennison and Pedersen (2012), Fallon, Park, and Yu (2015), Israelov, Nielsen, and Villalon (2017), and Israelov, Klein, and Tummala (2018).

8. Comparisons across strategies are not easy since both volatility and maximum drawdown are problematic risk measures for option strategies. We note that the top line has five times lower 5% cVaR (and volatility) than the lowest line. We studied several other strategies, including rolling the one-year puts only at expiry or buying long straddles (which try to isolate the volatility risk premium); none earned positive long-term returns.

9. Bhansali (2014) illustrates four techniques of active tail hedge management: monetization, extension, conversion, and rotation. Monetization, in its simplest form, involves liquidating the tail hedge (the previously purchased put) whenever its value hits an arbitrary multiple of its initial value (say, 5×) any time before expiry, buying a new OTM put (thereby reducing the protection compared to the just-sold put), and reinvesting the remaining proceeds into equities. Extension involves seeking opportunities across maturities (e.g., extending from near-expiry puts into relatively cheaper longer-dated puts after a crash when the term structure of volatility is inverted). Conversion technique could exchange direct purchase of puts for put spreads (the latter are cheaper amid high volatility). Rotation refers to the exchange of costly direct hedges in one market (S&P 500) for indirect hedges in other markets (say, options in credits, or even a trend-following strategy). The demystifying effort is commendable, but the usual caveats apply. Like any active management techniques, these have the potential to improve, but also to hurt, the performance of a static rolling put strategy. There is often an ex-ante tradeoff between cost and convexity in designs, and luck tends to trump skill in many ex-post outcomes.

10. See Ilmanen (2011, 2012) for a summary and references to various theories, as well as Bondarenko (2014), which argues "rational" arguments such as equilibrium models and "peso problems" fall short in explaining why puts are empirically overpriced.

11. Another strand of literature focuses on the question "Are low-probability, high-impact events underestimated or overestimated?" The Kahneman and Tversky (1979) prospect theory's decision weighting function suggests common overweighting of rare events (reflecting some mix of beliefs and preferences), while the concept of disaster myopia suggests that extremely rare events are ignored. Bordalo, Gennaioli, and Shleifer (2012) reconcile these

views by arguing that *salient* possibilities are less likely to be underweighted. The danger of losing a big part of your wealth seems salient, which would be consistent with the apparent richness of puts and the survey evidence of high expectations of crash probabilities in Goetzmann, Kim, and Shiller (2016).

12. See AQR Portfolio Solutions Group (2015, 2018), Nielsen, Thapar, and Villalon (2019), and McQuinn, Thapar, and Villalon (2021). These papers document the combined performance of various tail hedges and the underlying portfolio.

13. The previous section listed several reasons why perfect wealth floor protection is likely to prove elusive, especially at longer horizons. That said, any jackpot in a crash situation is valuable. If the investor faces inflexible spending needs, a monetized jackpot can provide the needed cash, and the investor does not have to sell risky assets at depressed prices. If there's no spending need, the investor can go bargain hunting. And the mere smoothing of portfolio returns over time reduces the variance drag in compounding wealth over time. All these features may give a tail hedge a valuable role in a portfolio even if its standalone expected return is mildly negative.

14. In practice, many investors will not take the naive Put strategy. They want something better from active tail risk managers, or simply do not hedge. The manager gets flexibility to try to reduce costs in good times without compromising reliability or convexity (the tail event payoff) too much, or vice versa. As noted earlier, we have insufficient data histories and limited transparency to judge active managers, and short-term performance variation is more likely to reflect luck than skill.

15. These behavioral biases are not specific to tail hedging strategies (see Goyal, Ilmanen, and Kabiller (2015)) but may apply especially to them due to the feast-and-famine payoff pattern.

Appendix

Data Descriptions

Trend is a hypothetical backtest based on trend-following investing which involves going long markets that have been rising and going short markets that have been falling, expecting that those trends over the examined look-back periods will continue. The strategy was constructed with an equal-weighted combination of one-month, three-month, and twelve-month trend-following strategies for 67 markets across 4 major asset classes: 29 commodities, 11 equity indices, 15 bond markets, and 12 currency pairs. We use futures returns when they are available. Prior to the availability of futures data, we rely on cash index returns financed at local short rates for each country. The strategy targets a long-term volatility of 10% but does not limit volatility during periods where realized volatility may be higher or lower than this number. All assets are weighted to have equal volatility, using the thiry-six-month rolling volatility over time.

The **CBOE Eurekahedge Tail Risk Index** is an equally weighted index of eight constituent funds. The index is designed to provide a broad measure of the performance of underlying hedge fund managers that specifically seek to achieve capital appreciation during periods of extreme market stress.

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Disclosure

AQR Capital Management is a global investment management firm that might or might not apply investment techniques or methods of analysis that are similar to those described in this article. The views expressed in this article are those of the authors and not necessarily those of AQR.